

## Trisect a Line Segment (6)

Method:

- 1) Construct segment AB
- 2) Construct segment AC on point A, but not-coincident with AB
- 3) Take an arbitrary point D on segment AC
- 4) Construct a circle at point D with radius AD.
- 5) Circle D intersects segment AC at E
- 6) Construct circle E with radius DE
- 7) Circle E intersects segment AC at point H
- 8) Construct segment BH
- 9) Construct segment EG that is parallel to segment BH
- 10) Construct segment DF that is parallel to segment BH
- 11) Segments DF and EG intersects segment AB at points F and G respectively.

*Claim: Points F and G trisect segment AB.*

Proof:

By construction, triangles ADF, AEG, and ABH are all similar. Furthermore,  $AD = \frac{1}{3}AH$  and  $AE = \frac{2}{3}AH$ . So

$$\Delta ADF = \frac{1}{3}\Delta ABH \text{ and } \Delta AEG = \frac{2}{3}\Delta ABH.$$

Hence  $AF = \frac{1}{3}AB$  and  $AG = \frac{2}{3}AB$ . Additionally, it implies  $GB = \frac{1}{3}AB$ .

$$AF + FG + GB = \frac{1}{3}AB + \frac{1}{3}AB + \frac{1}{3}AB = AB$$

Hence points F and G trisect segment AB.

